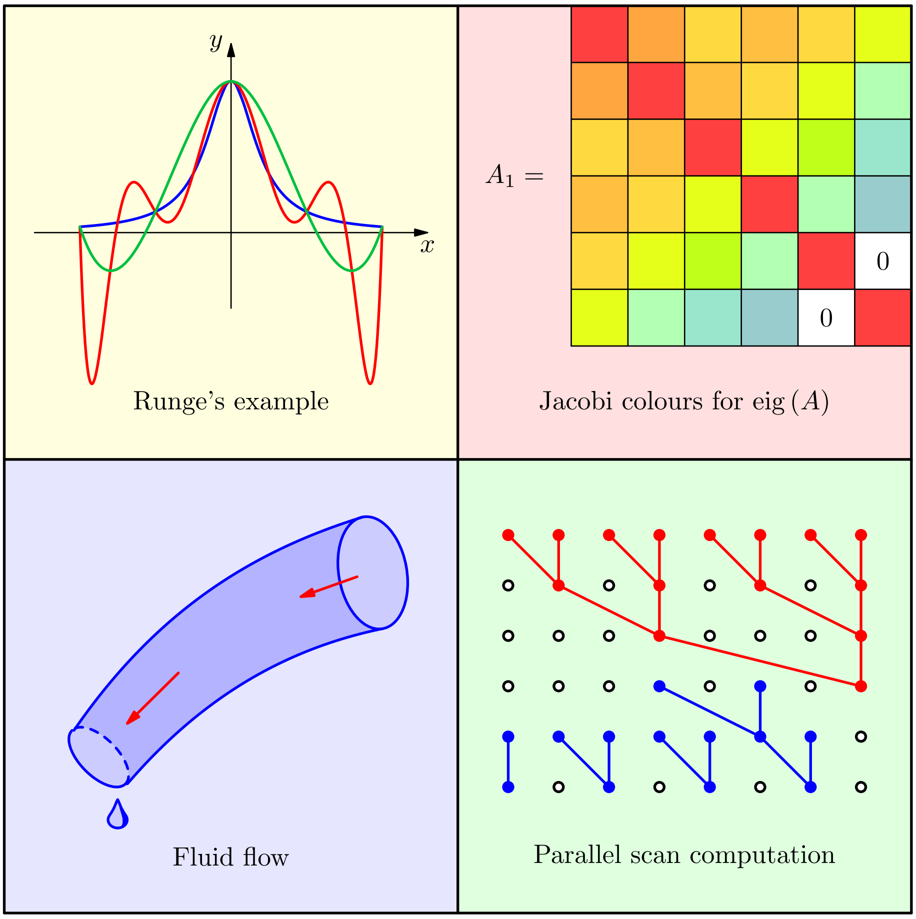
Conference on

# APPLIED MATHEMATICS
# AND
# SCIENTIFIC COMPUTING

## Šibenik, Croatia

## June 10–14, 2013




Runge's example

Jacobi colours for eig $(A)$

Fluid flow

Parallel scan computation

# SCIENTIFIC PROGRAM

## Monday, June 10 – Morning Session

| 8:20–8:30 | **Conference opening – Eduard Marušić–Paloka** |
|---|---|

| **Time** | **Chair: Eduard Marušić–Paloka** |
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| 8:30–9:15 | Gregory Panasenko, Ruxandra–Marina Stavre<br>Asymptotic analysis of a viscous non–steady flow–thin rigid plate interaction, page 11 |
| 9:15–9:40 | Andrijana Ćurković and Eduard Marušić–Paloka<br>Interaction of a thin fluid layer with an elastic plate, page 21 |
| 9:40–10:05 | Francisco Javier Suárez–Grau<br>On the Navier boundary condition for quasi–newtonian viscous fluids in rough domains, page 56 |

| **Time** | **Chair: Grigory Panasenko** |
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| 10:30–10:55 | Tomislav Fratrović and Eduard Marušić–Paloka<br>Brinkman's law for polymer fluids, page 30 |
| 10:55–11:20 | Neringa Klovienė and Konstantinas Pileckas<br>The second grade fluid flow problem in the infinite cylindrical domains, page 36 |
| 11:20–11:45 | Mindaugas Skujus<br>On the correct asymptotic conditions at infinity for the time-periodic Stokes problem set in a system of semi–infinite pipes, page 54 |

| **Time** | **Chair: Stephan V. Joubert** |
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| 11:55–12:20 | Irina Astashova<br>On power and non-power Kneser solutions to higher order nonlinear differential equations, page 15 |
| 12:20–12:45 | Luka Korkut, Domagoj Vlah, Darko Žubrinić and Vesna Županović<br>Fractal properties of oscillatory solutions of a class of ordinary differential equations, page 38 |
| 12:45–13:10 | Kseniya Dulina<br>Asymptotic stability of solutions to some classes of nonlinear second-order differential equations with parameters, page 23 |

## Monday, June 10 – Afternoon Session

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| 15:55–16:20 | Irina Eglite and Andrei Kolyshkin<br>Linear instability of shallow mixing layers with non-constant friction coefficient, page 25 |
| 16:20–16:45 | Sergey Zabolotskiy<br>Asymptotic equivalence of solutions to a generalization of the Lane–Emden equation, page 65 |

| Time | Chair: Andrei Kolyshkin |
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| 17:00–17:25 | Agnieszka Siluszyk<br>Relative equilibrium configurations in the planar $N$-body problem and their linear stability, page 53 |
| 17:25–17:50 | Eugene Bataev<br>Numerical solution of free boundary problem for American options pricing, page 16 |

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| 9:15–9:40 | Krešimir Burazin and Marko Vrdoljak<br>Homogenisation theory for Friedrichs systems, page 20 |
| 9:40–10:05 | Marko Vrdoljak<br>Variants of optimality criteria method for optimal design problems, page 63 |

| **Time** | **Chair: Marko Vrdoljak** |
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| 10:55–11:20 | Marin Mišur<br>On a generalization of compactness by compensation in the $L^p - L^q$ setting, page 45 |
| 11:20–11:45 | Rachad Zaki<br>Homogenization in periodic porous domains, page 66 |

| **Time** | **Chair: Alexey Filinovskiy** |
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| 12:20–12:45 | Maroje Marohnić and Josip Tambača<br>Derivation of the prestressed elastic rod model, page 43 |
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| 16:20–16:45 | Johanna C. Greeff, Quay Van der Hoff, and Temple H. Fay<br>Stability of a predator–prey model with predator population saturation, page 30 |

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| 9:15–9:40 | Brahim Amaziane, Mladen Jurak, Leonid Pankratov, and Anja Vrbaški<br>On the homogenization for some double porosity models for immiscible two-phase flow in a fractured reservoir, page 13 |
| 9:40–10:05 | Brahim Amaziane, Mladen Jurak, and Ana Žgaljić Keko<br>Numerical simulation of compositional compressible two–phase flow in porous media in global pressure formulation, page 14 |

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## Thursday, June 13 – Morning Session

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| 9:15–9:40 | Alexey Filinovskiy<br>On the estimates to the eigenvalues of the elliptic boundary value problem with a parameter, page 28 |
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| 11:20–11:45 | Zlatko Drmač<br>Recent developments in rational Krylov framework for $\mathcal{H}_2$ model order reduction, page 21 |

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| 12:45–13:10 | Nevena Jakovčević Stor and Ivan Slapničar<br>Accurate eigenvalue decomposition of rank-one modifications of diagonal matrices, page 33 |

## Excursion

## Friday, June 14 – Morning Session

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| 9:15–9:40 | Miljenko Marušić<br>Application of interpolatory exponentially fitted difference schemes to advection–diffusion problem, page 43 |
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| Time | **Chair: John J. H. Miller** |
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| 10:55–11:20 | Holger Rauhut, Reinhold Schneider and Željka Stojanac<br>Low–rank tensor recovery via iterative hard thresholding, page 48 |
| 11:20–11:45 | Nela Bosner<br>Balancing algorithms for three matrices, page 18 |

| Time | **Chair: Miljenko Marušić** |
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| 12:45–13:10 | Ilona M. Kulikovskikh and Sergei A. Prokhorov<br>Some lightweight algorithms for scientific computing in mobile technologies, page 40 |

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| 13:10–13:20 | **Conference closing – Eduard Marušić–Paloka** |

# INVITED LECTURES

WEDNESDAY, 8:30–9:15

## Multiphase multicomponent thermodynamically consistent formulation, using persistent primary variables, for numerical modeling

Alain Bourgeat

**Abstract.** Motivated by modelling the gas migration in an underground nuclear waste repository, we derive a multicompositional model of compressible multiphase flow and transport in porous media, with interphase mass transfer. One of the main difficulty appearing in the usual models is their inadequacy to take in account both fully and partially saturated situations, leading to numerical problems or unphysical numerical constraints. The new unified modeling of fully and partially saturated porous materials, presented in our talk, is based on fundamental principles of fluid mechanic and thermodynamic. Introducing "persistent" variables, like concentrations, instead of the traditional ones like saturation, makes possible a unified numerical treatment of fully and partially water saturated situations. Under adequate assumptions, the existence of solutions for equations corresponding to this "unified fully and partially saturated" formulation could be proved. We present numerical simulations , showing the efficiency of this modelling, related to the water-hydrogen flow in the vicinity of a nuclear waste repository.

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Wedneday, 15:30–16:15

## Characterization of electric fields in periodic composites

Marc Briane

**Abstract.** In this work, in collaboration with G.W. Milton and A. Treibergs from the University of Utah, we try to characterize among all the smooth periodic gradient fields those are isotropically realizable in the space or in the torus, i.e., are solutions of a conductivity equation with a suitable isotropic conductivity possibly periodic. When the gradient field does not vanish at some point, an isotropic conductivity can be built in the neighboring of this point thanks to the rectification theorem. More generally, using the gradient flow a sufficient condition for the global realizability in the space is that the gradient does not vanish. This condition is also necessary in dimension two but not in dimension three. However, the isotropic realizability does not hold if we impose in addition the periodicity of the conductivity. This is illustrated by a simple example. Then, we give a complete characterization of the isotropic realizability in the torus involving a condition on the gradient flow. On the other hand, the realizability of smooth periodic matrix–valued fields and laminate fields is also investigated. The realizability result is strongly based on the sign of the determinant of the matrix field.

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Tuesday, 8:30–9:15

## Some estimates for the pressure when its gradient is in $W^{-1,1}$ and applications in homogenization

Juan Casado Díaz

**Abstract.** Thanks to the Calderong–Zigmund estimates, it is well known that if a function $p$ defined on a smooth open set of $R^N$, has its gradient in $W^{-1,q}$, $q > 1$, then, it belongs to the space $L^q$. However the result does not hold for $q = 1$. In a joined paper with M. Briane, we use some results due to J. Bourgain and H. Brezis to prove that if $q = 1$ then $p$ is the sum of a function in $L^1$ and a distribution in $W^{-1,N}$.

The result applies to the homogenization of the Stokes and the membrane equations in dimension 2 with coefficients in $L^1$. It can also be applied to the homogenization of some non-newtonian problems in dimension $N$.

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Friday, 8:30–9:15

## Why are parameter-uniform numerical methods important?

John J. H. Miller

**Abstract.** The practical importance of singular perturbation problems is illustrated. The necessity for constructing special numerical methods for singular perturbation problems is explained. Parameter–uniform numerical methods for singular perturbation problems are introduced. The implementation of these methods and the mathematical analysis of their convergence properties are discussed. Sample numerical results are presented.

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Monday, 8:30–9:15

## Asymptotic analysis of a viscous non–steady flow–thin rigid plate interaction

Gregory Panasenko, Ruxandra–Marina Stavre

**Abstract.** We consider the special scaling for the elastic properties of a stratified rigid and heavy thin plate in contact with the moving fluid. Asymptotic expansion of a solution to this problem is constructed. In the special case when the plate doesn't move as a rigid body its leading term describes the model [1]. In the general case the limit problem describes the longitudinal motion of the plate as a rigid body coupled to the fluid motion. Some other possible scalings are discussed. The special high order junction conditions for 1D–2D approximation of the plate are asymptotically derived.

## References

[1] G. Panasenko, R. Stavre, *Asymptotic analysis of a periodic flow in a thin channel with visco-elastic wall*, J. Maths Pures Appl. **85** (2006) 558–579.

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Thursday, 8:30–9:15

# Analysis and approximation of Hamilton–Jacobi–Bellman equations with Cordès coefficients

Endre Süli

**Abstract.** I shall present the results of a series of recent joint papers with Iain Smears (University of Oxford), concerning the analysis and approximation of non-divergence-form linear second-order elliptic equations and the, fully nonlinear, elliptic Hamilton–Jacobi–Bellman equation, whose coefficients satisfy a Cordès condition. Stochastic analysis and stochastic control are fertile sources of partial differential equations of this kind.

We prove the existence of a unique strong solution and develop an *hp*-version discontinuous Galerkin finite element method for the elliptic Hamilton–Jacobi–Bellman equation with Cordès coefficients. We prove that the method is consistent and stable, and exhibits convergence rates that are optimal with respect to the mesh size $h$, and suboptimal in the polynomial degree $p$ by only half an order. Numerical experiments on problems with strongly anisotropic diffusion coefficients illustrate the high accuracy and computational efficiency of the scheme.

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# CONTRIBUTED TALKS

WEDNESDAY, 9:15–9:40

## On the homogenization for some double porosity models for immiscible two-phase flow in a fractured reservoir

Brahim Amaziane, Mladen Jurak, Leonid Pankratov, and Anja Vrbaški

**Abstract.** We consider immiscible two-phase flows through a fractured reservoir. This type of porous media consists of two porous structures with highly contrasted properties, a disconnected periodic set of blocks of usual porous media which are surrounded by a net of highly permeable fractures. We apply the homogenization theory to establish two models describing the global behavior of two–phase immiscible flow in fractured porous media. The first model deals with incompressible flow in a reservoir with thin fractures and the second one is a double porosity model of immiscible compressible flow in a global pressure formulation. We prove the convergence of the homogenization procedures using the two-scale convergence.

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Wednesday, 9:40–10:05

# Numerical simulation of compositional compressible two–phase flow in porous media in global pressure formulation

Brahim Amaziane, Mladen Jurak, and Ana Žgaljić Keko

**Abstract.** We consider liquid and gas flow in a porous medium with water and hydrogen component, taking into account capillary effects and diffusivity. The nonlinear system of partial differential equations that usually models this type of flow is transformed into a new system that employs an artificial variable called the global pressure and the total hydrogen mass density as main unknowns. The main advantage of the obtained model is that it does not require changing of main unknowns between saturated and unsaturated zones, which is not the case if the original system is used.

We apply a vertex centered finite volume method for the spatial and a fully implicit Euler method for the time discretization to solve the obtained system numerically. Numerical simulations are presented for one-dimensional test cases in a homogeneous porous medium.

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Monday, 11:55–12:20

# On power and non-power Kneser solutions to higher order nonlinear differential equations

Irina Astashova

**Abstract.** Straightforward calculations show that the equation

$$y^{(n)} = (-1)^n |y(x)|^k, \quad n \geq 2, \quad k > 1, \tag{1}$$

has the solutions

$$y(x) = C(x - x_0)^{-\alpha}$$

with the constants $\alpha = \frac{n}{k-1}$, $C^{k-1} = \alpha(\alpha+1)\cdots(\alpha+n-1)$, and arbitrary $x_0$.

It was proved for this equation with $n = 2$ (see [1]) and $3 \leq n \leq 4$ (see [2]) that all its Kneser solutions, i.e., those satisfying $y(x) \to 0$ as $x \to \infty$ and $(-1)^j y^{(j)}(x) > 0$ for $0 \leq j < n$, have the above power form. However, it was also proved (see [3]) that for any $N$ and $K > 1$ there exist an integer $n > N$ and $k \in \mathbb{R}$, $1 < k < K$, such that equation (1) has a solution

$$y(x) = (x - x_0)^{-\alpha} h(\log(x - x_0)),$$

where $h$ is a positive periodic non-constant function on $\mathbb{R}$.

Still it is not clear how large $n$ should be for existence of that type of solutions.

**Theorem.** *Suppose* $12 \leq n \leq 14$. *Then there exists* $k > 1$ *such that equation* (1) *has a solution* $y(x)$ *satisfying*

$$y^{(j)}(x) = (x - x_0)^{-\alpha-j} h_j(\log(x - x_0)), \quad j = 0, 1, \ldots, n-1,$$

*with periodic positive non-constant functions* $h_j$ *on* $\mathbb{R}$.

These results for $n = 12$ was proved earlier by I. Astashova and S. Vyun [4].
The research was supported by RFBR (grant 11–01–00989).

## References

[1] I. T. Kiguradze, T. A. Chanturia, *Asymptotic properties of solutions of nonautonomous ordinary differential equations*, Kluwer Academic Publishers, Dordrecht-Boston-London (1993).

[2] I. V. Astashova, *Qualitative properties of solutions to quasilinear ordinary differential equations*, in: I. V. Astashova (ed.) Qualitative Properties of Solutions to Differential Equations and Related Topics of Spectral Analysis: scientific edition, pp. 22–290. M.: UNITY-DANA (2012), (in Russian).

[3] V. A. Kozlov, *On Kneser solutions of higher order nonlinear ordinary differential equations*, Ark. Mat. **37** (1999), no. 2, 305–322.

[4] I. V. Astashova, S. A. Vyun, *On positive solutions with non-power asymptotic behavior to Emden–Fowler type twelfth order differential equation*, Differential equations **48** (2012), no. 11, 1568–1569, (in Russian).

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Monday, 17:25–17:50

## Numerical solution of free boundary problem for American options pricing

Eugene Bataev

**Abstract.** The paper describes a numerical algorithm that is the basis of software for vanilla put American options pricing.

We wish to solve the following problem: is it possible to evaluate the market price $P(S,t)$ of the American put option at time $t$, $0 \le t \le T$?

In papers [2, 3] it is proved that $P(S,t)$ is also the solution to the variational inequality, which is the weak form of the following set of inequalities:

$$\frac{\partial P}{\partial t} + \frac{\sigma^2(S,t)S^2}{2}\frac{\partial^2 P}{\partial S^2} + rS\frac{\partial P}{\partial S} - rP \le 0$$

$$\left(\frac{\partial P}{\partial t} + \frac{\sigma^2(S,t)S^2}{2}\frac{\partial^2 P}{\partial S^2} + rS\frac{\partial P}{\partial S} - rP\right)(P - P_0) = 0$$

$$P \ge P_0, P|_{t=T} = P_0.$$

Here $S(t)$ is the price of underlying asset at time $t$,

$$r(t) \in C^1 : [0,T] \to [0,+\infty]$$

is the price of risk-free asset,

$$\sigma(S,t) \in C^1 : [0,+\infty] \times [0,T] \to [0,+\infty]$$

is the volatility, $T$ is the maturity date, $K$ is the strike,

$$P_0(S,t) = (K - S)_+.$$

We prove that for $P(S,t)$ there exists $\gamma(t)$ dividing the region into two parts: where $P(S,t) = P_0(S)$, and where $P(S,t) > P_0(S)$. The function $\gamma(t)$ is called *the free boundary* or *the exercise boundary*.

On the basis of [1] we develop software which allows to find numerically the exercise boundary and the American option price.

## References


[1] Y. Achdou, O. Pironneau, *Computational methods for Option Pricing*, SIAM, Philadelphia, 2005, p. 316.

[2] A. Bensoussan, J. L. Lions, *Impulse Control and Quasivariational Inequalities*, Gauthier–Villars, 1984.

[3] P. Jailet, D. Lamberton, B. Lapeyre, *Variational inequalities and the pricing of American options*, Acta Applicandae Mathematicae **21** 1990, 263–289.



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Thursday, 11:55–12:20

# On the convergence of Jacobi methods under certain periodic pivot strategies


Erna Begović and Vjeran Hari



**Abstract.** Jacobi method for computing the eigensystem of a symmetric matrix $A$ is the iterative process of the form

$$A^{(k+1)} = U_k^* A^{(k)} U_k, \quad k \geq 0, \quad A^{(0)} = A,$$

where $U_k$, $k \geq 0$, are orthogonal matrices generated by the method. It is known that this process converges under some classes of cyclic pivot strategies and among them the class of weakly wavefront strategies is best known and understood.

We consider several new classes of periodic pivot strategies. They include certain cyclic and quasi–cyclic strategies. The convergence proofs use different tools and one of them is the theory of Jacobi operators introduced by Henrici and Zimmermann. The new strategies can also be used with block Jacobi methods, but then a new tool, the theory of block Jacobi operators, is used.

The obtained results and the new tools can be used for proving convergence of standard and block Jacobi–type methods for solving other eigenvalue and singular value problems.

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Friday, 11:20–11:45

## Balancing algorithms for three matrices

Nela Bosner

**Abstract.** Three versions of an algorithm for balancing three matrices simultaneously are proposed. The balancing is performed by premultiplication and postmultiplication with positive definite diagonal matrices, in order to reduce magnitude range of all elements in the involved matrices.

Some numerically stable algorithms applied to matrices with a wide range in the magnitude of the elements can produce results with large error. As an application we presented several problems from control theory. A reduction to the $m$-Hessenberg–triangular–triangular form is efficiently used for computing the frequency response

$$\mathcal{G}(\sigma) = C(\sigma E - A)^{-1}B + D$$

of a descriptor system. The reduction algorithm can produce inaccurate result for badly scaled matrices. Numerical experiments confirmed that balancing matrices $A$, $B$ and $E$ before the $m$-Hessenberg–triangular–triangular reduction can produce an accurate frequency response matrix. Balancing three matrices can also improve solution of the pole assignment problem in descriptor linear systems via state feedback. Another applications are quadratic eigenvalue problem $\lambda^2 Ax + \lambda Ex + Bx = 0$,

solution of the algebraic linear system $(\sigma^2 A + \sigma B + C)x = b$ for several values of the parameter $\sigma$, and any other problems involving three matrices of the same dimensions. These problems can be balanced with a simplified version of the proposed algorithm.

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http://web.math.hr/~nela

FRIDAY, 11:55–12:20

## Tension spline with application on image resampling

Tina Bosner, Bojan Crnković and Jerko Škifić

**Abstract.** Digital raster images often need to be represented in higher and lower resolutions. Resampling of digital images is an essential part of image processing. The most efficient and sufficiently accurate image resampling technics can produce spurious oscillations near sharp transitions of color. To improve that, we introduce the tension splines applied dimension by dimension.

The presented tension spline procedure provides an elegant solution to the image resampling by constructing a smooth approximation with sharp non-oscillatory resolution of discontinuities. The numerical results on real digital images are given to show effectiveness of the proposed algorithm.

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Tuesday, 9:15–9:40

## Homogenisation theory for Friedrichs systems

Krešimir Burazin and Marko Vrdoljak

**Abstract.** General homogenisation theory was originally developed for the stationary diffusion equation. Considering a sequence of such problems, with common boundary conditions, the homogenisation theory asks the question of what form is the limiting equation? The notions of $G$-convergence of corresponding operators, and $H$-convergence (also known as strong $G$-convergence) of coefficients were introduced. Later, the similar questions were studied for parabolic problems, linearized elasticity problems etc.

As Friedrichs systems can be used to represent various boundary value problems for (partial) differential equations, it is of interest to study homogenisation in such a wide framework, generalizing the known situations. Here we introduce concepts of G and H-convergence for Friedrichs systems, give compactness theorems under some compactness assumptions, and discuss some other interesting topics, such as convergence of adjoint operators, topology of H-convergence and possibility for appearance of nonlocal effects.

Finally, we apply this results to the stationary diffusion equation, the heat equation, the linearized elasticity system, and a model example of first order equation leading to memory effects. In the first three cases, the equivalence with the original notion of H-convergence is proved. Here the Quadratic theorem of compensated compactness is used in order to verify our compactness assumptions.

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Monday, 9:15–9:40

## Interaction of a thin fluid layer with an elastic plate

Andrijana Ćurković and Eduard Marušić–Paloka

**Abstract.** We consider a viscous incompressible fluid interaction with an elastic plate located on one part of the fluid boundary. The flow is modelled by Stokes equations. The plate displacement is modelled by the linear plate theory for transverse motions of purely elastic plate. Due to the small displacement of the plate, we neglect the deformation of the fluid domain. We study the dynamics of this coupled fluid–structure system in the limit when the thickness of the fluid layer tends to zero. A set of effective equations is obtained. The approximation is justified through a weak convergence result. Results on uniqueness and regularity of solution of the effective equations are obtained.

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Thursday, 11:20–11:45

## Recent developments in rational Krylov framework for $\mathcal{H}_2$ model order reduction

Zlatko Drmač

**Abstract.** The Iterative Rational Krylov (IRKA) algorithm for model order reduction (Gugercin, Antoulas, Beattie 2008.) has recently attracted attention because of its effectiveness in real world applications, as well as because of its mathematical elegance. The key idea is to construct a reduced order model that satisfies a set of necessary (Meier–Luenberger) optimality conditions formulated as interpolatory conditions: the reduced $r$-th order transfer function interpolates the full $n$-th order

function and its derivative(s) in the reflected (about the imaginary axis) images of the reduced order poles. This is formulated as a fixed point problem, and the interpolation nodes are generated as $\sigma^{(k+1)} = \phi(\sigma^{(k)})$. Here $\phi(\cdot)$ computes the eigenvalues of the Petrov–Galerking projection of the state matrix to rational Krylov subspaces computed at $\sigma^{(k)} = (\sigma_1^{(k)}, \ldots, \sigma_r^{(k)})$.

The most expensive part of the IRKA (and other algorithms that implement the Meier–Luenberger approach as interpolatory procedure) is computing the transfer function at a sequence of dynamically generated points in the right half–plane $\mathbb{C}_+$. This is in particular complicated if the system to be reduced is given in a state-space realization $(A, B, C)$ and evaluating the transfer function at $\sigma$ involves solution of shifted linear system with the coefficient matrix $A - \sigma I$. (In some cases, a realization independent implementation is possible and transfer function can be obtained directly from the PDE model, thus bypassing the state space realization.) This setting is not suitable for a data driven framework, since we are given only the transfer function values along the imaginary axis.

On the other hand, if we interpret the necessary optimality conditions as orthogonality (in $\mathcal{H}_2$) of the residual to the tangent space at the reduced order model to the manifold of the models of given reduced order, we can deploy rich theory of the geometry of linear systems, as well as reproducing kernel space property of $\mathcal{H}_2$. All action takes place on $\mathrm{i}\mathbb{R}$, and, with a suitable kernel, function evaluation can be achieved by a linear functional, implemented using numerical quadrature. This new framework is well suited for data driven applications. It also offers efficient implementation and deeper insight in the behavior and properties of the numerical algorithm.

This is a joint work with C. Beattie and S. Gugercin from Virginia Tech, Blacksburg, USA.

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Monday, 12:45–13:10

# Asymptotic stability of solutions to some classes of nonlinear second-order differential equations with parameters

Kseniya Dulina

**Abstract.** We consider the question of asymptotic stability of zero solution to the equation

$$y'' + 2\sigma\mu y' - \bigl(k^2\mu^2 - \mu\varphi(\tau)\bigr)\sin y = 0, \quad \sigma > 0, \quad k > 0, \quad \mu \approx 0, \tag{1}$$

where $\varphi(\tau)$ is an arbitrary continuous $T$-periodic function, $\mu$, $k$, $\sigma$ are parameters.

The inversed pendulum equation of motion is a special case of equation (1) when the pivot point movement is described by an arbitrary $T$-periodic function.

**Theorem** *Let $\varphi(\tau) = \varphi(\tau + T)$,*

$$\int\limits_0^T \varphi(\tau)\, d\tau = 0,$$

*where $\varphi(\tau)$ is a continuous function, and*

$$k^2 < -\left(\frac{1}{T}\int\limits_0^T \tau\varphi(\tau)\, d\tau\right)^2 - \frac{1}{T}\int\limits_0^T\int\limits_0^\tau (T-\tau_1)\varphi(\tau_1)\, d\tau_1\ \varphi(\tau)\, d\tau.$$

*Then for sufficiently small parameters $\mu > 0$ the zero solution to the equation (1) is asymptotically stable.*

By using the results from [1, 2, 3] we obtain the attraction domains and establish estimates of the exponential decay of solutions to the equation (1). The above results are valid in the case of piecewise continuous function $\varphi(\tau)$ having a finite number of simple discontinuities.

## References

[1] G. V. Demidenko, I. I. Matveeva, *On stability of solutions to quasilinear periodic systems of differential equations*, Siberian Math. J. **45** (2004), no. 6, 1271–1284.

[2] G. V. Demidenko, I. I. Matveeva, *On stability of solutions to linear systems with periodic coefficients*, Siberian Math. J. **42** (2001), no. 2, 332–348.

[3] G. V. Demidenko, I. I. Matveeva, *On asymptotic stability of solutions to nonlinear systems of differential equations with periodic coefficients*, Selcuk J. Appl. Math. **3** (2002), no. 2, 37–48.

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Wednesday, 11:55–12:20

## A discrete exterior calculus model of blood flow in the retina

Andrea Dziubek, Giovanna Guidoboni, Anil Hirani, and Edmond Rusjan

**Abstract.** Discrete exterior calculus (DEC) has the potential to enhance the existing numerical methods in continuum mechanics, especially in the study of fluid flow on curved surfaces and in the elasticity of rods and shells. A rich source of problems, where DEC promises to have an edge, are biomedical applications. This talk has two goals: first, we present a general framework for the study of continuum mechanics problems on curved surfaces with DEC, and second, we use this framework to model blood flow in the retina of the eye for the study of glaucoma.

We translate the vector calculus model to exterior calculus on curved embedded manifolds. Using exterior calculus the familiar vector calculus operators: gradient, divergence and curl can be expressed in a coordinate free notation in terms of the exterior derivative, the wedge product and the Hodge star operator. DEC uses the concept of a dual mesh to discretize these operators and reduces partial differential equations to matrix equations. The form of the discrete exterior derivative is uniquely determined by the Stokes theorem. The discretization of the Hodge star operator is more subtle and still an open question. Currently, the most promising approaches are the circumcentric dual mesh with the discrete Hodge operator determined by the ratio of primal and dual cell sizes and the barycentric dual mesh with Whitney forms.

Second, we apply the framework to the blood flow in the eye. We solve Darcy flow equations for non-homogeneous porous media on a flat circular surface and on a

hemispherical surfaces for blood pressure and velocity. Four major blood vessels are used as sources. The various sizes of the blood vessels, from arterioles to capillaries to venules, are modeled by an additional abstract dimension.

Glaucoma is one of the leading causes of blindness. It develops slowly, without symptoms. The loss of vision is caused by the damage to the retinal ganglion neurons and their fibers in the optic nerve, which are vulnerable to injury due to pressure. Many details about the progression of the disease remain unknown. Our study shows that alterations in ocular curvature, such as those occurring in myopic eyes, might contribute to glaucomatous damage by reducing retinal blood flow.

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Monday, 15:55–16:20

## Linear instability of shallow mixing layers with non-constant friction coefficient

Irina Eglite and Andrei Kolyshkin

**Abstract.** Linear instability of shallow mixing layers is analyzed in many papers where bottom friction is modeled by the Chezy or Manning formulas. Friction coefficient in such cases is assumed to be constant. There are cases, however, where resistance force is not constant with respect to the transverse coordinate. One

important practical application includes flows in vegetated areas (this situation often occurs during floods). The difference between resistance forces in vegetated area and main channel should be taken into account for proper modeling of the flow.

The flow is described by a system of shallow water equations under the rigid–lid assumption. Introducing the stream function and linearizing the resulting equation in the neighborhood of the base flow we obtain the linear stability problem. It is assumed that the friction coefficient is not constant in the transverse direction. The variability of the friction coefficient with respect to the transverse coordinate is described by the formula

$$c_f(y) = c_{f_0}\gamma(y),$$

where $c_{f_0}$ is constant and $\gamma(y)$ is an arbitrary differentiable shape function. The linear stability problem is solved by the method of normal modes. Numerical solution of the corresponding eigenvalue problem is obtained by means of a collocation method based on Chebyshev polynomials. Critical values of the bed-friction number are obtained for different values of the parameters of the problem. Results of numerical calculations are compared with the case of a constant friction coefficient.

An asymptotic scheme based on the method of multiple scales is suggested in order to analyze the behavior of the most unstable mode in a weakly nonlinear regime where the bed-friction number is slightly smaller than the critical value.

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Tuesday, 10:30–10:55

## Comparison of H-measures and semiclassical measures

Marko Erceg

**Abstract.** Semiclassical measures were introduced by Patrick Gérard (1991), but are also known under the name of Wigner measures as Pierre Louis Lions and Thierry Paul gave a different construction using the Wigner transform. They are mathematical objects used in the study of high–frequency limits in continuum and quantum mechanics. In contrast to the H-measures, they are tailored to deal with problems which have a characteristic length (e.g. thickness of a plate).

Recently, Luc Tartar extended semiclassical measures to a functional on continuous functions on a compactification of $\mathbf{R}^d \setminus \{0\}$. Our aim is to study more deeply this extension and its relation to H-measures. It is known that for an $\varepsilon_n$-oscillatory sequence we can obtain that the corresponding H-measure and the semiclassical measure are equal, but for the above extension such assumption is superfluous, while the proof is simpler.

In particular, we shall study PDEs with characteristic length tending to zero to emphasize situations where H-measures and semiclassical measures do not give the same information, and where with semiclassical measures we indeed get something more.

This is a joint work with Nenad Antonić and Martin Lazar.

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Wednesday, 12:20–12:45

## Interaction of water waves with a submerged rough disc a using spectral method

Leandro Farina

**Abstract.** The problem of solving a hypersingular integral equation over a nonplanar disc is considered. Using a perturbation method, the problem is reformulated by a sequence of simpler hypersingular equations over a flat unitary disc making it well suited for an efficient spectral method. The first order approximation is computed and the hydrodynamic force due to heaving radiation motion is presented in terms of the added mass and damping coefficients for a polynomial cap and for a rough disc. The latter is modelled by a superposition of sinusoidal surfaces defined by randomly generated parameters. The solution exhibits larger maxima associated with smaller volume of submergence. Rough discs with similar statistical properties exhibit different behaviours. Thus, it is the exact specific form of the rough disc that dictates the hydrodynamic force.

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Tuesday, 12:45–13:10

## The Van Der Pol equation: an energy perspective

Temple H. Fay

**Abstract.** We discuss in some small detail the well known non-dissipative van der Pol oscillator equation,

$$\ddot{x} + a(b - x^2)\dot{x} + x = 0,$$

a popular example having a limit cycle. In doing so on a Test Model, we provide template for finding the important features of the limit cycle. In particular, we produce an energy plot by obtaining a relationship between the sum of kinetic and potential energies and displacement alone, bypassing velocity. We calculate maximum amplitudes of oscillations without solving the equation.

*Mathematica* is the computer algebra system used but any good ODE solver package would suffice. The energy plot shows exactly how and when energy is being injected into and dissipated from the model.

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Thursday, 9:15–9:40

## On the estimates to the eigenvalues of the elliptic boundary value problem with a parameter

Alexey Filinovskiy

**Abstract.** We consider the spectral problem

$$\sum_{i,j=1}^{n} \left(a_{ij}(x)u_{x_i}\right)_{x_j} + \lambda u = 0, \quad x \in \Omega, \quad (u_N + \alpha g(x)u)\,\big|_{\Gamma} = 0, \tag{1}$$

in a bounded domain $\Omega \subset \mathbb{R}^n$, $n \geq 2$, with the smooth boundary $\Gamma$. Coefficients $a_{ij}(x)$, $i, j = 1, \dots, n$, of the differential operator

$$Lu = \sum_{i,j=1}^{n} (a_{ij} u_{x_i})_{x_j}$$

are real-valued functions from $C^1(\overline{\Omega})$ satisfying the symmetry condition $a_{ij} = a_{ji}$ and ellipticity condition

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \theta \sum_{i=1}^{n} \xi_i^2, \quad \theta > 0, \quad x \in \Omega, \quad (\xi_1, \dots, \xi_n) \in \mathbb{R}^n,$$

$$u_N = \sum_{i,j=1}^{n} a_{ij} u_{x_i} \nu_j,$$

where $\nu = (\nu_1, \dots, \nu_n)$ is the outer unit normal vector to $\Gamma$. Function $g(x)$ is a continuous function on $\Gamma$ satisfying the condition $g(x) \geq g_0 > 0$, $x \in \Gamma$, $\alpha$ is a real parameter.

Denote by $\lambda_1(\alpha)$ the first eigenvalue of the problem (1). Let $\lambda_1^{(d)}$ and $\tilde{u}$ are the first eigenvalue and eigenfunction of the Dirichlet spectral problem

$$Lu + \lambda u = 0, \quad x \in \Omega, \quad u \Big|_{\Gamma} = 0.$$

**Theorem 1.** *For $\alpha > 0$ the following inequality holds:*

$$\lambda_1(\alpha) \geq \left( \left(\lambda_1^{(d)}\right)^{-1} + \left(\alpha q_g\right)^{-1} \right)^{-1},$$

*where*

$$q_g = \inf_{\substack{y \in H^1(\Omega) \\ Ly=0}} \left( \int_\Gamma g y^2 \, ds \Big/ \int_\Omega y^2 \, dx \right).$$

**Theorem 2.** *For $\alpha \to +\infty$ the following asymptotic expansion holds:*

$$\lambda_1(\alpha) = \lambda_1^{(d)} - \frac{\int_\Gamma g^{-1} \tilde{u}_N^2 \, ds}{\int_\Omega \tilde{u}^2 \, dx} \alpha^{-1} + o(\alpha^{-1}).$$

We also study various properties of eigenvalues

$$\lambda_k(\alpha) = \sup_{v_1, \dots, v_{k-1} \in L_2(\Omega)} \inf_{\substack{v \in H^1(\Omega) \\ (v, v_j)_{L_2(\Omega)} = 0, \\ j = 1, \dots, k-1}} \left( \int_\Omega \sum_{i,j=1}^{n} a_{ij} v_{x_i} v_{x_j} \, dx + \alpha \int_\Gamma g v^2 \, ds \right) \Big/ \int_\Omega v^2 \, dx,$$

for $k = 2, 3, \dots$, of the problem (1) for $-\infty < \alpha < +\infty$ and obtain the estimates to $\lambda_k(\alpha)$ as $\alpha \to +\infty$.

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Monday, 10:30–10:55

## Brinkman's law for polymer fluids

Tomislav Fratrović and Eduard Marušić–Paloka

**Abstract.** A stationary non-Newtonian fluid flow through the periodic porous medium is observed and investigated by the methods of asymptotic analysis. Filtration law for a polymer fluid obtained by the homogenization process is a nonlinear Brinkman–type law in case of the critical obstacle size. Derivation of the Brinkman's law is done by Γ-convergence of corresponding energy functionals.

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Tuesday, 16:20–16:45

## Stability of a predator–prey model with predator population saturation

Johanna C. Greeff, Quay Van der Hoff, and Temple H. Fay

**Abstract.** To confirm the existence of limit cycles in two–dimensional differential systems is a non-trivial problem involving advanced mathematical techniques. A simple yet effective technique to investigate the possible existence (or non-existence) of bounded solutions to dynamical systems, primarily predator–prey models, of the

form $x = f(x, y)$ and $y = g(x, y)$ is proposed. This tool, referred to as the Invariant Region Method, is a general technique that can be applied to confirm the existence of an invariant region – if such a bounded region does exist, then Poincaré–Bendixson theory can be applied and the study of the stability of the model reduces to simple eigenvalue analysis.

In addition, a novel predator–prey model that supersedes most existing predator–prey models, is proposed. This model, referred to as the FGH model, incorporates not only the usual logistic growth term for the prey, but also introduces a predator population saturation term. The proposed model is robust and results in system stability for a large range of parameter values and is applicable to various fields of sciences where mathematical models are used to describe the dynamics of the system under investigation.

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Wednesday, 17:00–17:25

## On the determination of resonance parameters by means of analytical continuation

Jiří Horáček

**Abstract.** In order to determine resonance parameters the Schrödinger equation

$$-\frac{d^2\psi_l(r)}{dr^2} + \frac{l(l+1)}{r^2}\psi_l(r) + V(r)\psi_l(r) = E\psi_l(r),$$

where $E = k^2$, has to be solved subject to the boundary conditions of Siegert

$$\psi_l(0) = 0, \quad \frac{\psi_l'(R)}{\psi_l(R)} = ik,$$

$R$ being a distant point where the interaction $V(r)$ can be neglected. The eigenvalue $E$ determined in this way is in general complex,

$$E = E_R - \frac{i}{2}\Gamma,$$

and the solution $\psi$ not normalizable. This makes the problem of determination of resonance parameters very difficult. To avoid this problem we introduce an attractive perturbation $U(r)$ multiplied by a free parameter $\lambda$ (coupling constant)

$$V(r) \to V(r) + \lambda U(r)$$

and apply the bound state type boundary condition on the solution, i.e., $\psi(r \to \infty) = 0$ instead of the Siegert condition. Doing this we eliminate the eigenvalue from the boundary condition and arrive at standard normalizable solution. At increasing $\lambda$ the new potential $V(r)$ gets more attractive and some resonance states transform into bound states. The idea is to solve the bound state problem for several values of the parameter $\lambda$, which is an easy task, to obtain the bound state energies as functions of $\lambda$ and then extrapolate $\lambda \to 0$ to obtain the resonance energy. This idea was proposed in the field of nuclear physics by Krasnopolsky and Kukulin. It was shown that the extrapolation must be replaced by an analytic continuation in a new variable $\sqrt{\lambda - \lambda_0}$ where $\lambda_0$ is the branch point of the function $E(\lambda)$. The analytical continuation is performed by using the statistical (multipoint) Padé approximation.

It will be shown that although this approach is ill-conditioned the use of the statistical Padé approximation allows us to obtain stable and accurate values of the continued parameters provided the input data are of sufficient accuracy.

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THURSDAY, 12:45–13:10

## Accurate eigenvalue decomposition of rank-one modifications of diagonal matrices

Nevena Jakovčević Stor and Ivan Slapničar

**Abstract.** We present a new algorithm for solving an eigenvalue problem for a real symmetric matrix which is a rank-one modificiation of a diagonal matrix. The algorithm computes all eigenvalues and all components of the corresponding eigenvectors with high relative accuracy in $O(n^2)$ operations. The algorithm is based on a shift-and-invert approach. Only a single element of the inverse of the shifted matrix eventually needs to be computed with double of the working precision. Each eigenvalue and the corresponding eigenvector can be computed separately, which makes the algorithm adaptable for parallel computing. Our results extend to Hermitian case. The method can be used as a part of divide-and-conquer method for real symmetric tridiagonal matrices.

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TUESDAY, 11:55–12:20

## Nonlinear anisotropic damping and Bryan's effect

Stephan V. Joubert and Michael Y. Shatalov

**Abstract.** Bryan's effect [1] is the phenomenon of the rotation of the vibration pattern within the body of a rotating, vibrating body. For an ideal symmetric structure, the rotation rate of the pattern is proportional to the inertial rotation rate of the body. Indeed, in 1890 Bryan showed (for an ideal ring or bell) that the

constant of proportionality $\eta$ (known nowadays as Bryan's factor) is given by the formula

$$\eta = \frac{\text{Rate of rotation of the vibrating pattern}}{\text{Inertial rate of rotation of the vibrating structure}}.$$

Vibratory gyroscopes [2] that are used for deep–space missions [3], measure the rotation rate of the pattern and, with a known value for Bryan's factor $\eta$, any rotation of the space-craft can be determined. For a slow inertial rotation of the vibrating body, it has been demonstrated that the rate of rotation of the damped vibration pattern is the same as it would be for an ideal rotating vibrating body if the damping is assumed to be *isotropic, not necessarily linear* [4, 5]. The inclusion of *anisotropic linear* damping into the equations of motion of a slowly rotating, vibrating, spherical body has been demonstrated in [6]. In this paper we demonstrate how to include *anisotropic, not necessarily linear* damping into the equations of motion of a slowly rotating, vibrating, (hemi)spherical body and illustrate that the rotation rate of the vibrating pattern is no longer proportional to the inertial rotation rate – this will seriously compromise the accuracy of a vibratory gyroscope based on Bryan's effect. The resulting system of nonlinear differential equations facilitate the discussion of possible control methods to limit the affects of anisotropic nonlinear damping.

## References


[1] G. H. Bryan, *On the beats in the vibrations of a revolving cylinder or bell*, Proc. Cambridge Philos. Soc. Math. Phys. Sci. **7**, (1890), 101–114.

[2] E. J. Loper, D. D. Lynch, *Hemispherical Resonator Gyroscope*, U.S. Patent No. 4,951,508, 1990. Available online: http://www.patentbuddy.com/Patent/4951508, accessed January 2013.

[3] D. M. Rozelle, *The hemispherical resonator gyro: From wineglass to the planets*, in Proceedings of the 19th AAS/AIAA Space Flight Mechanics Meeting, Savannah, Georgia, 2009, pp. 1157–1178. Available online: http://132.228.182.15/by_division/navigationsystems/whitepapers/assets/hrg.pdf, accesed January 2013.

[4] M. Y. Shatalov, S. V. Joubert, C. E. Coetzee and I. Fedotov, *Free vibration of rotating hollow spheres containing acoustic media*, J. Sound and Vibration **322** (2009) no. 4–5, 1038–1047.

[5] S. V. Joubert, M. Y. Shatalov and A. V. Manzhirov, *Bryan's effect and isotropic nonlinear damping*, submitted.

[6] M. Y. Shatalov, S. V. Joubert and C. E. Coetzee, *Analysing manufacturing imperfections in a spherical vibratory gyroscope*, in Proceedings of the 4th IEEE International Workshop on Advances in Sensors and Interfaces (IWASI), Borgo Egnazia, Savelletri di Fasano, Brindisi, Italy, June 28–29, 2011, pp. 165–170. Available online: DOI: 10.1109/IWASI.2011.6004665, accessed January 2013.

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Wednesday, 16:15–16:40

# Nonhomogeneous boundary value problem for the stationary Navier–Stokes system in unbounded domains

Kristina Kaulakytė and Konstantinas Pileckas

**Abstract.** We consider nonhomogenous boundary value problem for the stationary Navier–Stokes system

$$\begin{cases} -\nu\Delta\mathbf{u} + \big(\mathbf{u}\cdot\nabla\big)\mathbf{u} + \nabla p = 0 & \text{in } \Omega,\\ \operatorname{div}\mathbf{u} = 0 & \text{in } \Omega,\\ \mathbf{u} = \mathbf{a} & \text{on } \partial\Omega \end{cases}$$

in domain $\Omega$ with outlets to infinity. The boundary $\partial\Omega$ is multiply connected and consists of connected noncompact components, forming the outer boundary, and connected compact components, forming the inner boundary. We suppose that the fluxes over the components of the inner boundary are sufficiently small, while we do not impose any restrictions on fluxes over the infinite components of the outer boundary. Note that the total flux of sources and sinks is equal to zero. Depending

on the geometry of outlets to infinity, the Dirichlet integral of the solution may be either finite or infinite.

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MONDAY, 10:55–11:20

## The second grade fluid flow problem in the infinite cylindrical domains

Neringa Klovienė and Konstantinas Pileckas

**Abstract.** We consider the second grade fluid flow problem

$$\begin{cases} \dfrac{\partial}{\partial t}(\mathbf{u}-\alpha\Delta\mathbf{u})-\nu\Delta\mathbf{u}+\operatorname{curl}(\mathbf{u}-\alpha\Delta\mathbf{u})\times\mathbf{u}+\nabla\widetilde{p}=\mathbf{f}, \\ \operatorname{div}\mathbf{u}=0, \\ \mathbf{u}\big|_{S^T}=0, \quad \mathbf{u}(x,0)=\mathbf{u}_0(x), \\ \int_\sigma u_n dx'=F(t) \end{cases}$$

in three different unbounded domains:

- the two-dimensional channel,
- the three–dimensional axially symmetric pipe,
- the three–dimensional pipe with an arbitrary cross section.

In the first two cases the existence of a unique unidirectional Poiseuille type solution is proved and the relation between the flux of the velocity field and the gradient of the pressure is found.

The analogous results are obtained for the time periodic problem

$$\begin{cases} \dfrac{\partial}{\partial t}(\mathbf{u}-\alpha\Delta\mathbf{u})-\nu\Delta\mathbf{u}+\operatorname{curl}(\mathbf{u}-\alpha\Delta\mathbf{u})\times\mathbf{u}+\nabla\widetilde{p}=\mathbf{f}, \\ \operatorname{div}\mathbf{u}=0, \\ \mathbf{u}\big|_{S^T}=0, \quad \mathbf{u}(x,0)=\mathbf{u}(x,2\pi), \\ \int_\sigma u_n dx'=F(t), \quad F(0)=F(2\pi) \end{cases}$$

in the two-dimensional channel and in the three–dimensional axially symmetric pipe.

It is proved that in the three-dimensional pipe with an arbitrary cross section the unidirectional solution does not exists even if data are unidirectional. However, for sufficiently small data in this case exists a unique solution having all three components $\big(u_1(x_1,x_2,t),\ u_2(x_1,x_2,t), u_3(x_1,x_2,t)\big)$ of the velocity field $\mathbf{u}$ and the velocity components $\big(u_1,u_2\big)$ perpendicular to the $x_3$-axis of the cylinder are secondary comparing with $u_3$.

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Thursday, 10:30–10:55

## Calculation of complex eigenvalues for axisymmetric problems in eddy current testing

Valentina Koliskina and Inta Volodko

**Abstract.** Eddy current method is often used to test quality of electrically conducting materials. If a flaw is present in a conducting medium then the parameters of the flaw are estimated solving the inverse problem where the difference (in some norm) between experimental and theoretical data is minimized. Thus, reliable and accurate method for the solution of a direct problem is needed in order to solve the inverse problem.

Quasi-analytical solutions for problems where a conducting medium is of finite size can be constructed by truncated eigenfunction expansion method. One of the important steps in the solution procedure is the computation of complex eigenvalues. Eigenvalue problem of the form

$$\varphi(z) = 0$$

where $z$ is complex arises when boundary conditions at the interface between media with different properties are satisfied. There are two important aspects of the solution of equation $\varphi(z) = 0$:

(a) no initial guesses for the roots are available and

(b) a relatively large number of roots (up to 70) has to be computed.

In the present paper we consider the solution of the eigenvalue problem for the following three cases:

(1) a coil with alternating current above a conducting cylinder of finite size,

(2) a coil above a conducting plate with bottom cylindrical hole and

(3) a coil above a conducting half-space with a flaw in the form of a cylinder of finite size.

Computational algorithm is described and results of numerical computations of eigenvalues are presented for all three cases mentioned above.

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Monday, 12:20–12:45

## Fractal properties of oscillatory solutions of a class of ordinary differential equations

Luka Korkut, Domagoj Vlah, Darko Žubrinić and Vesna Županović

**Abstract.** The fractal oscillatority of solutions $x = x(t)$ of ordinary differential equations at $t = \infty$ is measured by *oscillatory* and *phase dimensions* defined through the box dimension. Oscillatory and phase dimensions are defined as box dimensions of the graph of $X(\tau) = x(1/\tau)$ near $\tau = 0$ and trajectory $(x, \dot{x})$ in $\mathbb{R}^2$, respectively, assuming that $(x, \dot{x})$ is a spiral converging to the origin. The box dimension of a plane curve measures the accumulation of a curve near a point, which is in particular interesting for non-rectifiable curves. The oscillatory dimension of solutions of Bessel equation has been determined by Pašić and Tanaka (2011). Here, we compute the phase dimension od solutions of a class of ordinary differential equations, including Bessel equation. The phase dimension of solutions of Bessel equation has been computed to be equal to 4/3. We also compute the phase dimension of a class of $(\alpha, 1)$-chirp-like functions, related to Bessel equation, to be equal to $2/(1 + \alpha)$. We determined the box dimension of a specific type of spirals that we called *wavy*

*spirals*, which are converging to the origin, but not with a nonincreasing radius function.

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Thursday, 10:55–11:20

## Efficient eigenvalue estimation for parametrized problems in the reduced basis method

Daniel Kressner and Petar Sirković

**Abstract.** The reduced basis (RB) method provides a powerful framework for the solution of parametrized partial differential equations (PDEs). It consists of an *offline* stage, where solutions of the PDE belonging to suitably chosen parameter values are computed and collected in a (low-dimensional) subspace. This subspace is computed once and for all. The subsequent *online* stage then uses a Galerkin approach based on the subspace only. This may speed up the solution process dramatically, especially if the PDE needs to be solved for many parameter values.

For simplicity, let us consider the variational formulation of a parametrized elliptic PDE:

$$\text{Find } u \equiv u(\mu) \in H \text{ such that } a(u, v; \mu) = f(v; \mu) \text{ for all } v \in H,$$

where $H$ is a Hilbert space and $\mu$ represents one or several parameters. As it is common in RB methods, we assume that $a$ admits an affine linear decomposition

$$a(u, v; \mu) = \Theta_1(\mu) a_1(u, v) + \cdots + \Theta_q(\mu) a_q(u, v).$$

*A posteriori* error analysis is an important part of the RB method to ensure its reliability. This requires to obtain tight lower bounds or the coercivity constant $\alpha(\mu)$ of $a(\cdot, \cdot, \mu)$ or, equivalently, for the smallest eigenvalue of a symmetric eigenvalue problem $A(\mu) - \lambda M$. For this purpose, the so called Successive Constraint Method (SCM) has become quite popular. It makes use of the smallest eigenvalues at (hopefully not too many) parameter samples $\mu_1, \ldots, \mu_J$ to yield satisfactory lower and upper bounds elsewhere. Note that the computation of these smallest eigenvalues amounts to the solution of $J$ (discretized) PDE eigenvalue problems. When using a preconditioned method, like LOBPCG, for this purpose, it turns out that further information (e.g., the second/third smallest eigenvalues and the corresponding eigenvectors) can be determined at an almost negligible additional cost. Several ideas on using this information to improve the performance of SCM will be discussed in the talk.

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Friday, 12:45–13:10

## Some lightweight algorithms for scientific computing in mobile technologies

Ilona M. Kulikovskikh and Sergei A. Prokhorov

**Abstract.** This article is motivated by the rapid development of mobile technologies in computer science. A particular feature of mobile technologies is portability and small size in contrast to high–performance technologies. However, this fact determines the minimal requirements for scientific computing, such as low run-time, user-friendly interface and limited memory. The main purpose of this research is to create a lightweight algorithms according to these requirements. In addition, a required accuracy of research results must be provided, that is necessary to create the competitive service for scientific computing.

The lightweight algorithms are based on Fourier series expansion method:

$$\hat{f}(x) = \sum_{k=0}^{m} \hat{\beta}_k \psi_k(x),$$

where $\psi_k(x)$ is an orthogonal polynomials, that is defined on $[0, x_{\max}]$ with the weight function $\mu(x)$ and the norm $\|\psi_k\|^2$; $\hat{\beta}_k$ is the Fourier's coefficients, which is estimated by means of a numerical–analytical approach to provide the required accuracy of research results,

$$\hat{\beta}_k = \frac{1}{\|\psi_k\|^2} \sum_{i=0}^{I_{\max}-1} \left( a_i \int_{x_i}^{x_{i+1}} \psi_k(x)\mu(x)\,dx + b_i \int_{x_i}^{x_{i+1}} x\psi_k(x)\mu(x)\,dx \right),$$

where $a_i, b_i$ are the linear interpolation coefficients according to

$$f(x) = \sum_{i=0}^{I_{\max}-1} \left(a_i + b_i x\right)\delta_i.$$

However, we offer to extend Fourier series expansion method to solve the problem in the following way: to determine the functional transformation associated with the orthogonal polynomials $\vartheta_k(x)$, such as

$$\int \psi_k(\tau,\gamma)\mu(x)\,dx, \quad \int \tau\psi_k(x)\mu(x)\,dx$$

as follows:

$$\vartheta_k(x) = \sum_{\nu=0}^{\infty} \alpha_{k,\nu}\psi_\nu(x), \quad \alpha_{k,\nu}\|\psi_\nu\|^2 = \int_0^{x_{\max}} \vartheta_k(x)\psi_\nu(x)\mu(x)\,dx.$$

In this case we have an extended orthogonality relation

$$\int_0^{x_{\max}} \vartheta_k(x)\psi_\nu(x)\mu(x)\,dx = h_{k,\nu}(\gamma), \quad k,\nu = 0,\dots,K,$$

which in contrast to the basic orthogonality relation has additional diagonals. But the matrix $H$ contains the finite number of the diagonals that makes possible to calculate the $\vartheta_k(x)$ without the additional numerical errors. As a result, we can calculate one matrix with the values of $\psi_k(x)$, what is located in computer memory, and the values of $\vartheta_k(x)$ can be determined by means of the above matrix.

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Tuesday, 17:00–17:25

# Correlated equilibrium in bimatrix games with fuzzy payoffs

Zoltán Makó

**Abstract.** In game theory, a correlated equilibrium is a solution concept that is more general than the well known Nash equilibrium. It was first discussed by Aumann (1974). The idea is that each player chooses his action according to his observation of the value of the same public signal. A strategy assigns an action to every possible observation a player can make. A distribution $\pi$ is defined to be a correlated equilibrium if no player can ever expect to unilaterally gain by deviating from his recommendation, assuming the other players play according to their recommendations. The set of correlated equilibria is a convex polytope, described by finitely explicit linear inequalities.

In many practical problems, the quantities can only be estimated. In the case when the quantities are coefficients of the bimatrix games, they may be characterized with fuzzy numbers. In this talk we consider bimatrix games with fuzzy payoffs. For any pair of strategies, a player receives a payoff represented as a quasi–triangular fuzzy number. For example, when a payoff matrix of a game is constructed by information from a competitive system, elements of the payoff matrix would be ambiguous if imprecision or vagueness exists in the information.

In practice, when one or more coefficients of the optimization problem have uncertain values, then the optimal value will be uncertain. In order to reach the always $\alpha$-level of optimal value we must take an optimal decision. Although the optimal value is uncertain, the decision must be unambiguous. Therefore, $\alpha$-optimal solution set contains vectors of real numbers. The concepts of modified joint optimal solution and fuzzy optimal value defined by Makó (2006) do comply with the above presented requirements. These concepts are founded on the notion of joint optimal solution defined by Buckley 1995).

In this talk we aim at utilizing these concepts to define the correlated equilibrium for a bimatrix game with fuzzy payoffs by using possibility distributions approach.

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Tuesday, 12:20–12:45

## Derivation of the prestressed elastic rod model

Maroje Marohnić and Josip Tambača

**Abstract.** We derive a one-dimensional model for the displacement of a longitudinally stressed elastic rod starting from a cylindrical three-dimensional linearized prestressed elastic body with a small diameter. The prestress is due to the prior elastic deformation of an isotropic, homogenous, elastic body. We deduce the scaling of forces by a formal asymptotic expansion. Then we prove that the family of solutions of three-dimensional problems converges to a limit that is the unique solution of the prestressed rod model. The energy of the limit model is a sum of the classical bending energy of the elastic rod and the membrane energy of the prestressed string.

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Friday, 9:15–9:40

## Application of interpolatory exponentially fitted difference schemes to advection–diffusion problem

Miljenko Marušić

**Abstract.** We consider a class of previously proposed exponentially fitted difference schemes and apply them to singularly perturbed boundary value problem of the form:

$$\varepsilon y'' + by' = f.$$

Since difference schemes are derived from the interpolation formulae for exponential sum, they are consistent. Stability is proved for three-point schemes. For four–point schemes we analyze behavior of the approximation error in the limit case when $\varepsilon \to 0$. Considered methods are stable in the limit case.

## References


[1] D. Braess, *Nonlinear Approximation Theory*, Springer-Verlag, Berlin, 1986.

[2] M. K. Kadalbayo, K. C. Patidar, *A survey of numerical techniques for solving singularly perturbed ordinary differential equations*, Appl. Math. Comput. **130** (2002), 457–510.

[3] M. Marušić, *Limit Properties of interpolation by exponential sums*, Proceedings of BAIL 2002, Perth, Australia, July 8–12, 2002, (eds. S. Wang and N. Fowkes), University of Western Australia, (2002) pp. 183–188.

[4] ——, *A family of exponentially fitted difference schemes for singularly perturbed two-point boundary value problems based on interpolation*, manuscript (2012).

[5] ——, *Four-point interpolatory exponentially fitted difference schemes for singularly perturbed two-point boundary value problems*, manuscript (2012).

[6] J. J. H. Miller, E. O'Riordan, and G. I. Shishkin, *Fitted Numerical Methods for Singular Perturbation Problems*, World Scientific, Singapore, 1996.

[7] H. G. Roos, M. Stynes, L. Tobiska, *Numerical Methods for Singularly Perturbed Differential Equations*, Springer–Verlag, New York, 1996.



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Thursday, 12:20–12:45

## On accuracy of Jacobi methods


Josip Matejaš and Vjeran Hari



**Abstract.** Using a new subtle error analysis we have proved sharp relative error bounds for the most important two-sided Jacobi methods. They solve the symmetric eigenvalue problem, the singular value problem and the generalized eigenvalue problem, and are known by the names Jacobi, Kogbetliantz and Falk and Langemeyer. The aim of this communication is to present an overview of these results.

First, we briefly present the novelties in the error analysis: the terms of higher order in the machine precision are not neglected, and the signs of the errors are

taken into account. Using such approach the error bounds can be improved, especially because one has better control of intermediate errors during the computation. Typically, one can monitor and estimate the suppression and cancelation of the errors.

The first result refers to the relative accuracy of the Jacobi method on scaled diagonally dominant indefinite symmetric matrices. The next several results prove the relative accuracy of the Kogbetliantz method for triangular matrices. Here, the standard formulas for the diagonalization of a two by two triangular matrix had to be modified in order to obtain a relatively accurate algorithm. We have first considered the case when the initial matrix is scaled diagonally dominant. Later, we have considered the case when the initial matrix is a general triangular matrix. The most recent results include the relative accuracy of the Falk–Langemeyer method for the positive definite generalized eigenvalue problem $Ax = \lambda Bx$. Numerical tests illustrate and confirm the obtained accuracy results.

In addition, we have improved the relative perturbation bounds for the eigenvalues of scaled diagonally dominant Hermitian matrices and for the singular values of symmetrically scaled diagonally dominant square matrices.

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Tuesday, 10:55–11:20

## On a generalization of compactness by compensation in the $L^p - L^q$ setting

Marin Mišur

**Abstract.** We investigate conditions on two sequences $(u_r)$ and $(v_r)$ weakly converging to $u$ and $v$ in $L^p(\mathbf{R}^d;\mathbf{R}^N)$ and $L^{p'}(\mathbf{R}^d;\mathbf{R}^N)$ respectively, under which a quadratic form

$$q(x, u_r, v_r) = \sum_{j,m=1}^{N} q_{jm}(x) u_{jr} v_{mr}$$

converges toward $q(x,u,v)$ in the sense of distributions. A set of sufficient conditions involve fractional derivatives and variable coefficients, both in quadratic form and differential constraints, and they represent a generalisation of known compactness by compensation theory. The proofs are accomplished using recently introduced $H$-distribution concept.

Compactness by compensation theory (pioneered by F. Murat and L. Tartar) appeared to be a very useful tool in investigating problems involving partial differential equations (linear or nonlinear). We shall present an application to a linear equation of a parabolic type.

This is joint work with Darko Mitrović, University of Montenegro.

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Wednesday, 11:20–11:45

## A constructive existence proof for a moving boundary fluid–structure interaction problem

Boris Muha and Sunčica Čanić

**Abstract.** Motivated by modeling blood flow in human arteries, we study a fluid-structure interaction problem in which the structure is composed of multiple layers, each with possibly different mechanical characteristics and thickness. In the problem presented in this talk the structure is composed of two layers: a thin layer modeled by the 1D wave equation, and a thick layer modeled by the 2D equations of linear elasticity. The flow of an incompressible, viscous fluid is modeled by the Navier-Stokes equations. The thin structure is in contact with the fluid, thereby serving as a fluid-structure interface with mass. The coupling between the fluid and the structure is nonlinear. The resulting problem is a nonlinear, moving-boundary problem of parabolic-hyperbolic-hyperbolic type.

We will show that the model problem has a well-defined energy, and that the energy is bounded by the work done by the inlet and outlet dynamic pressure data. The spaces of weak solutions reveal that the presence of a thin fluid–structure interface with mass regularizes solutions of the coupled problem. We will present the main steps of a constructive proof of an existence of a weak solution for the considered problem. Theoretical results will be illustrated with numerical examples.

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WEDNESDAY, 10:55–11:20

## Brinkman–type models in fluid film lubrication

Igor Pažanin

**Abstract.** The aim of this talk is to present recent results on new asymptotic models for fluid film lubrication. We consider the situation in which two rigid surfaces being in relative motion are separated by a thin layer of fluid acting as a lubricant. Lower surface is assumed to be perfectly smooth, while the upper is rough with roughness described by some function $h$. Such situation appears naturally in many engineering applications consisting of moving machine parts, e.g., in journal bearings or computer disk drives. The lubricant is first assumed to be classical Newtonian fluid, and then we extend our analysis to incompressible micropolar fluid taking into account the fluid microstructure as well. Using asymptotic analysis with respect to the film thickness, we derive the higher–order corrections of the standard Reynolds approximation. The effective equations are similar to the Brinkman model for porous medium flow.

This is a joint work with Eduard Marušić–Paloka and Sanja Marušić.

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Tuesday, 15:30–15:55

## Necessary $p$-th order optimality conditions for irregular Lagrange problem in calculus of variations

Agnieszka Prusińska and Alexey A. Tret'yakov

**Abstract.** The contribution is devoted to singular calculus of variations problems with constraints which are not regular mappings at the solution point, i.e., its derivatives are not surjective.

We pursue an approach based on the constructions of the $p$-regularity theory. For $p$-regular calculus of variations problem we present necessary conditions for optimality in singular case and illustrate our results by classical example of calculus of variations problem.

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Friday, 10:55–11:20

## Low–rank tensor recovery via iterative hard thresholding

Holger Rauhut, Reinhold Schneider and Željka Stojanac

**Abstract.** It has been noticed that many types of real–world signals and images have a sparse expansion in terms of suitable basis or frame (for example wavelet expansion). Compressive sensing is the process of acquiring and reconstructing a signal that is supposed to be sparse or compressible from fewer linear measurements. Taking $m$ linear measurements of the signal $\mathbf{x} \in \mathbb{R}^N$ corresponds to applying measurement matrix $\mathbf{A} \in \mathbb{R}^{m \times N}$

$$\mathbf{y} = \mathbf{A}\mathbf{x}.$$

Since we are interested in the case where $m \ll N$, without further information recovery of the signal $\mathbf{x}$ is impossible. However, by assuming that the vector $\mathbf{x}$ is

$s$-sparse (i.e., $\|\mathbf{x}\|_0 = \#\{i : x(i) \neq 0\} \leq s$) the situation changes. Hence, main problem of the compressive sensing is solving the NP-hard $\ell_0$-minimization problem

$$\min_{\mathbf{z} \in \mathbb{R}^N} \|\mathbf{z}\|_0 \text{ such that } \mathbf{A}\mathbf{z} = \mathbf{y}.$$

Many reconstruction algorithms like $\ell_1$-minimization, LASSO, Iterative hard thresholding algorithm (IHT) have been introduced for recovering sparse signals.

Given a matrix $\mathbf{X} \in \mathbb{R}^{n_1 \times n_2}$ of rank at most $r \ll \min\{n_1, n_2\}$, the goal of the low-rank matrix recovery is to reconstruct $\mathbf{X}$ from linear measurements

$$\mathbf{y} = \mathcal{A}(\mathbf{X}),$$

where

$$\mathcal{A} : \mathbb{R}^{n_1 \times n_2} \to \mathbb{R}^m$$

with $m \ll n_1 n_2$.

We go one step further and consider recovery of low–rank tensors of order $d$ ($\mathbf{X} \in \mathbb{R}^{n_1 \times n_2 \times \cdots \times n_d}$) from a small number of measurements. A version of the Iterative hard thresholding algorithm (TIHT) for the higher order singular value decomposition (HOSVD) will be introduced. As a first step towards the analysis of the algorithm, we define a corresponding tensor restricted isometry property (HOSVD-TRIP) and show that Gaussian and Bernoulli random measurement ensembles satisfy it with high probability. In addition, with a reasonable extra condition on the rank-$\mathbf{r}$ approximations of given tensors, we are able to analyze the algorithm entirely.

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MONDAY, 15:30–15:55

# On a stochastic regularization of strong singular solutions to conservation laws

Olga Rozanova

**Abstract.** It is well-known that the density component of the pressureless gas dynamics system of conservation laws that pretends to describe a formation of structures in the Universe, basically contains strong delta–singularities. In 1D case this model was studied in numerous works from different points of view, nevertheless, the natures of the strong singularities, especially in multi D case is far from to be established.

We show that the pressureless gas dynamics system in an arbitrary dimension is a companion system for the stochastically perturbed Hopf equation for the velocity vector $V(t,x)$. This allows to use the technique of the representation of solution by means of integrals of the probability density function. Namely, we find the solution to the Fokker–Planck equation for the joint probability density of position and velocity of particle and prove that provided the solution $V$ to the Hopf equation keeps smoothness, the velocity $u(t,x)$ and density $\rho(t,x)$ of the companion conservation laws can be found by means of special formulae as a zero limit of stochastic perturbation. In particular, the method allows to describe asymptotically how starting from smooth data a $\delta$-singularity arises in a component of density.

Nevertheless, after the moment of singularity formation in the solution to the Hopf equation the form of the companion system changes. Namely, it becomes a gas dynamics system with a specific pressure term. This term corresponds in some sense to a polytropic one-atomic gas. The density component for this model has no delta-singularity and can contain only usual shock waves.

Further, to get a solution to the presureless gas dynamic system itself, we do an additional step based on a variational principle. This procedure can be easily performed numerically.

The method has an advantages comparing the method of viscous regularization of the Hopf equation even before the moment of the singularity formation. Indeed, in the multidimensional case the Cole–Hopf transform allows to reduce the problem to the heat equation and therefore to get the integral representation only for potential initial velocity. Our method allows to obtain the integral representation to the Cauchy problem for an arbitrary initial velocity.

This work is a continuation of [1].

## References

[1] S. Albeverio, A. Korshunova and O. Rozanova, *A probabilistic model associated with the pressureless gas dynamics*, http://www.sciencedirect.com/science/article/pii/S0007449713000572, see also e-print arXiv:0908.2084 and arXiv:1010.0212.



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Wednesday, 17:25–17:50

## Adaptive analytic discretization for Schrödinger operators


Andreas Ruffing



**Abstract.** Differential equations which contain the parameter of a scaling process are usually referred to by the name Quantum Difference–Differential Equations. Some of their applications to discrete models of the Schrödinger equation are presented and some of their rich, filigrane und sometimes unexpected analytic structures are revealed.

A Lie–algebraic concept for obtaining basic adaptive discretizations is explored, generalizing the concept of deformed Heisenberg algebras by Julius Wess. They are also related to algebraic foundations of quantum groups in the spirit of Ludwig Pittner.

Some of the moment problems of the underlying basic difference equations are investigated. Applications to discrete Schrödinger theory are worked out and some spectral properties of the arising operators are presented, also in the case of Schrödinger operators with basic shift–potentials and in the case of ground state difference–differential operators.



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Tuesday, 17:25–17:50

# On operator equilibrium problems

Júlia Salamon

**Abstract.** The equilibrium theory provides a unified, natural, innovative and general framework for the study of a large variety of problems such as optimization problems, fixed points problems, variational inequalities, Nash equilibria, saddle point problems and complementarity problems as special cases. The problems mentioned above often occur in mechanics, physics, finance, economics, network analysis, transportation and elasticity.

Domokos and Kolumbán [1] introduced the technique of working with operator solutions instead of scalar or vector variables in field of variational inequalities. Inspired by their work, Kum and Kim developed the scheme of operator variational inequalities from the single–valued case into the multi–valued one. The weak operator equilibrium problems were studied by Kazmi and Raouf [2], Kum and Kim.

The goal of this paper is to study the existence of solution for operator equilibrium problems. The problems under consideration are the following:

Let $X$, $Y$ and $Z$ be Hausdorff topological vector spaces; $L(X,Y)$ be the space of all continuous linear operators from $X$ to $Y$ and let $K \subset L(X,Y)$ be a nonempty convex set. Let $C : K \rightrightarrows Z$ be a set–valued mapping such that for each $f \in K$, $C(f)$ is a proper, solid, convex, closed cone with the apex at the origin of $Z$. Let a vector–valued mapping $F : K \times K \to Z$ be given.

The weak operator equilibrium problem *(OEP)* is to find $f \in K$ such that

$$F(f,g) \notin -\operatorname{Int} C(f), \quad \forall g \in K.$$

The strong operator equilibrium problem *(SOEP)* is to find $f \in K$ such that

$$F(f,g) \in C(f), \quad \forall g \in K.$$

## References

[1] A. Domokos and J. Kolumbán, *Variational inequalities with operator solutions*, J. Global Optim. **23** (2002), pp. 99–110.

[2] K. R. Kazmi and A. Raouf, *A class of operator equilibrium problems*, J. Math. Anal. Appl. **308** (2005), pp. 554–564.

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Monday, 17:00–17:25

# Relative equilibrium configurations in the planar $N$-body problem and their linear stability

Agnieszka Siluszyk

**Abstract.** In the works of Andoyer, Meyer, MacMillan and Bartky, Subbotin, Moulton and Brumberg [1] have presented an analysis of the existence the relative equilibrium configurations in the planar problem of four bodies in a barycentric frame of reference. By a *central configuration* in a barycentric frame of reference of $N$ material points $(m_i, q_i)$, $m_i \in \mathbb{R}^+$, $q_i = (x_i, y_i) \in \mathbb{R}^2$, $i = 1, \ldots, N$ we understand a configuration such that the total Newtonian acceleration on every body is proportional with the position vector of this body with respect to the center of mass of the configuration $\ddot{q}_i = \sigma \cdot q_i$, for some $\sigma \neq 0$ (see e.g. [2]). The central configurations are interesting because they allow to obtain explicit homographic solutions of the $N$-body problem. Central configurations also appear as a key point when we study the topology of the fibers in the phase space with fixed energy and angular momentum i.e., the fibers of the energy–momentum mapping (see [2]). Moreover above authors have investigated the stability of the solutions of the four material points in some special cases. We extend their results by adding numerical calculations for investigating four body problem, their stability by using CAS "Mathematica".

## References

[1] W. D. Macmilian, W. Bartky, *Configurations in the problem of four bodies*, Trans. Amer. Math. Soc. **34** (1932) 838–875.

[2] V. I. Arnold, V. V. Kozlov , A. I. Neishtadt *Mathematical Aspects of Classical and Celestial Mechanics*, Springer–Verlag, Berlin, 2006.

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Monday, 11:20–11:45

## On the correct asymptotic conditions at infinity for the time-periodic Stokes problem set in a system of semi–infinite pipes

Mindaugas Skujus

**Abstract.** We consider the time-periodic Stokes problem in domains with cylindrical outlets to infinity. It is well known that this problem may have infinitely many solutions. Prescribing the time-periodic flow rates through cross-sections of the outlets one can obtain a solution which is unique up to a time-dependent function in a pressure term. Our aim is to obtain some other physically reasonable conditions at infinity that lead to a well-posed problem. For this purpose we derive the generalized Green's formula and construct a special basis in the set of solutions to the homogeneous problem.

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Friday, 9:40–10:05

## One difference scheme for numerical solving of advection equation with aftereffect

Svyatoslav I. Solodushkin

**Abstract.** Let us consider an advection equation with time delay of general form

$$\frac{\partial u}{\partial x} + a\frac{\partial u}{\partial t} = f(x, t, u(x, t), u_t(x, \cdot)), \tag{1}$$

here $x \in [0; X]$, $t \in [t_0; \theta]$, $u(x, t)$, $u_t(x, \cdot) = \{u(x, t + \xi),\ -\tau \leq \xi < 0\}$ are a space and a time independent variables, unknown function and a prehistory–function of the unknown function to the moment $t$ respectively, $a > 0$. Initial $u(t, x) = \varphi(x, t)$, $x \in [0; X]$, $t \in [t_0 - \tau; t_0]$, and boundary $u(0, t) = g(t)$, $t \in [t_0; \theta]$, conditions are set. Consensual conditions are satisfied $g(t_0) = \varphi(0, t_0)$.

Let $N$ and $M$ be the number of partition points for $[0;X]$ and $[t_0;\theta]$, respectively. The uniform grid can be constructed $\{t_j,x_i\}_{j=0}^{M}{}_{i=0}^{N}$, where $t_j = t_0 + j\Delta$, $j = 0,\dots,M$, and $x_i = ih$, $i = 0,\dots,N$. Let $u_j^i$ denotes approximate solution $u(x_i,t_j)$ and $\varepsilon_j^i = u(x_i,t_j) - u_j^i$, $i = 0,\dots,N$, $j = 0,\dots,M$.

For $0 \le s \le 1$ we consider a family of grid methods

$$\begin{aligned} \tfrac{u_{j+1}^1-u_j^1}{\Delta} + a\left(s\,\tfrac{-4u_{j+1}^0-2h/a(f_{j+1}^0-\dot g_{j+1})+4u_{j+1}^1}{2h} + (1-s)\,\tfrac{-4u_j^0-2h/a(f_j^0-\dot g_j)+4u_j^1}{2h}\right) &= f_j^1,\\ \tfrac{u_{j+1}^i-u_j^i}{\Delta} + a\left(s\,\tfrac{u_{j+1}^{i-2}-4u_{j+1}^{i-1}+3u_{j+1}^i}{2h} + (1-s)\,\tfrac{u_j^{i-2}-4u_j^{i-1}+3u_j^i}{2h}\right) &= f_j^i, \end{aligned} \tag{2}$$

$i = 2,\dots,N$, $j = 0,\dots,M-1$. Here $f_j^i$ denotes value of $f$ in the node $(i,j)$,

$$\dot g_j = \frac{dg(t)}{dt}\Big|_{t=t_0+j\Delta}.$$

**Definition.** *Method converges with order $h^p + \Delta^q$, if there exists constant $C$, that*

$$\|\varepsilon_j^i\| \le C(h^p + \Delta^q)$$

*for all $i = 0,\dots,N$ and $j = 0,\dots,M$.*

**Theorem.** *Let the exact solution $u(x,t)$ of the problem (1) thrice continuously differentiable by $x$ and twice continuously differentiable by $t$, first derivative of the solution by $x$ is continuously differentiable by $t$. Then if $s > 0.5$ the method (2) converge with order $h^2 + \Delta$.*

Numerical experiments support the theoretical statements.

Supported by RFBR (grant No. 13–01–00089) and UrFU program for young scientists.

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Wednesday, 10:30–10:55

## High order approximations for incompressible viscous flow on a rough boundary

Maja Starčević and Eduard Marušić–Paloka

**Abstract.** In this talk we propose high order approximations of incompressible viscous fluid flow, needed for obtaining wall-laws of high order. We consider the two dimensional fluid flow in a channel with a rough side, modeled by the incompressible Stokes system. The roughness is periodic and the ratio of amplitude of the rough part and the rest of the flow domain is denoted by $\epsilon$, being a small number. We impose periodic boundary conditions on the flow. We generalize the boundary layers that we build into approximations and discuss the existence of the layers and their features. Finally we give the error estimates for the approximations of velocity and pressure.

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Monday, 9:40–10:05

## On the Navier boundary condition for quasi–newtonian viscous fluids in rough domains

Francisco Javier Suárez–Grau

**Abstract.** We consider the study of the asymptotic behavior of stationary flows of an incompressible quasi-newtonian fluid near a rough boundary $\Gamma_\varepsilon$ periodic with period $\varepsilon$ and amplitude $\delta_\varepsilon$ such that $\delta_\varepsilon \ll \varepsilon$.

We assume that the fluid satisfies on $\Gamma_\varepsilon$ the slip condition given by Navier's law

$$u_\varepsilon \cdot \nu = 0, \quad (S(\mathbb{D}[u_\varepsilon]) \cdot \nu + \gamma u_\varepsilon)_\tau = 0 \quad \text{on } \Gamma_\varepsilon,$$

where $u_\varepsilon$ is the velocity of the fluid and $\gamma \geq 0$ is the friction coefficient. The viscous stress tensor $S = S(\mathbb{D})$, $S : \mathbb{R}^{3\times 3}_{sym} \to \mathbb{R}^{3\times 3}_{sym}$, is assumed to be a function of the symmetric velocity gradient

$$\mathbb{D}[u_\varepsilon] = \left(Du_\varepsilon + D^T u_\varepsilon\right)/2,$$

and to satisfy $p$-coercivity and $(p-1)$-order growth for a certain $p \geq 2$, together with continuity, strictly monotonicity and the following homogeneity condition

$$S(t\mathbb{D}) = |t|^{p-2} t\, S(\mathbb{D}), \quad \forall\, \mathbb{D} \in \mathbb{R}^{3\times 3}_{\rm sym} \quad \text{and} \quad t \in \mathbb{R}.$$

We show that if $\delta_\varepsilon \gg \varepsilon^{\frac{2p-1}{p}}$ then Navier's law implies adherence condition in the limit. This justify mathematically the use of the adherence condition on rough domains. It doesn't happen the same in the cases $\delta_\varepsilon \sim \varepsilon^{\frac{2p-1}{p}}$ and $\delta_\varepsilon \ll \varepsilon^{\frac{2p-1}{p}}$.

## References


[1] D. Bucur, E. Feireisl and S. Necasová, *Influence of wall roughness on the slip behaviour of viscous fluids*, Proc. Roy. Soc. Edinburgh Sect. A **138** (2008), 957–973.

[2] J. Casado–Díaz, M. Luna–Laynez, and F. J. Suárez–Grau, *Asymptotic behavior of a viscous fluid with slip boundary conditions on a slightly rough wall*, Math. Models Methods Appl. Sci. bf 20 (2010), 121–156.



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Friday, 10:30–10:55

# The spectrum of speeds of topological solitons in singularly perturbed Hamiltonian systems

Ksenia Sukmanova

**Abstract.** We consider the following generalization of the nonlinear wave equation as the basic model:

$$l^4 u_{xxxx} + l_0^2 \left( u_{xxxx} - \frac{1}{c^2} u_{tt} \right) + f(u) = 0. \tag{1}$$

For solutions of the form $u(x,t) = u(x - vt)$, eq. (1) can be written in the form

$$\varepsilon u_{xxxx} + u_{xx} + f(u) = 0. \tag{2}$$

In the case when $f(u)$ is a piecewise linear function of the form

$$f(u) = \begin{cases} -\dfrac{1}{a} u, & u \in [0, a), \\ -\dfrac{2-u}{2-a}, & u \in [a, 4-a), \\ -\dfrac{1}{a}(u-4), & u \in [4-a, 4]) \end{cases} \tag{3}$$

Equation (2) is a Hamiltonian system with Hamiltonian

$$H = p_1 q_1 - \frac{1}{2\varepsilon^2} p_2^2 - \frac{1}{2} q_2^2 + F(q_1),$$

where

$$q_1 = u, \quad q_2 = u, \quad p_1 = u' + \varepsilon^2 u''', \quad p_2 = -\varepsilon^2 u'', \quad F'(u) = f(u).$$

Let us consider the problem on the topological soliton for eqs. (2)–(3) with the conditions

$$\lim_{x \to -\infty} = 0, \quad \lim_{x \to +\infty} = 4.$$

If $\varepsilon$ is small than eq. (2) is singularly perturbed. For this reason, results computed with numerical integration method are compared with results obtained by analytical method for join of solution.

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Tuesday, 15:55–16:20

# High order optimality conditions for p-regular inequality constrained optimization problems

Ewa Szczepanik and Alexey A. Tret'yakov

**Abstract.** In this talk, the conjugate cone is described for the general singular case in a constrained optimization problem with p-regular constraints and Kuhn–Tucker-type optimality conditions are constructed with the use of p-factor-operators for the general mathematical programming problem as described in [1]. Previously, necessary optimality conditions were proposed for some classes of optimization problems with irregular inequality constraints. For example, the case of a singularity of only up to the second order was considered or the higher derivative operators of the constraint functions were assumed to have nontrivial p-kernel, etc. In other cases, optimality conditions were written in the form of conjugate cones with no description.

## References

[1] V. G. Karmanov, D. V. Denisov, and A. A. Tret'yakov, *Simulation and analysis in decision making problems*, in Vychisl. Tsentr Ross. Akad. Nauk, Moscow, **16** (2003) 16–24, (in Russian).

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Wednesday, 12:45–13:10

# A comparison of the FVPM and FVM for compressible flows in time–dependent flow domains

Delia Teleaga, Marius Stoia–Djeska, and Luiza Zavalan

**Abstract.** This work deals with the computation of compressible flows with shocks in domains with moving boundaries such as those occurring in fluid-structure interaction and in many other industrial applications. Thus, in this paper we present two typical computational methodologies, which have been developed for the computation of unsteady compressible flows inside domains with complex and changeable geometries. The newer mesh-less Finite-Volume-Particle Method (FVPM) and the classical Finite Volume Method (FVM) with a moving mesh are both used to solve the Euler equations of gas-dynamics for unsteady flows with shocks and moving boundaries. The primary goal of this work is to compare the accuracy and efficiency of the two methods.

The physical problem taken under consideration in this work is a very simple fluid-structure interaction problem. The flow domain consists in a one-dimensional tube with constant cross-section closed by a flexible piston at the left end and by a fixed wall at the other end. The fluid-structure interaction problem is initiated either by moving suddenly the piston with an initial velocity or by a complex of shock and expansion waves due to a Riemann problem initiated at the middle of the tube. Because the actual position of the piston determines at least partially the fluid domain boundaries, it becomes necessary to perform the solution of the flow problem on a time dependent flow domain and thus:

a) the fluid solver deals with compressible flows with shocks and
b) the fluid–structure coupling strategy is based on similar time advancement strategies.

In the FVPM the weak formulation of the Euler system is discretized by restricting it to a discrete set of test functions. The test functions are chosen from a partition of unity with smooth and overlapping partition functions (the particles) which can move along prescribed velocity fields. For the hyperbolic case under consideration, the flow variables exchange between the fluid particles is based on the classical upwind numerical techniques. Geometrical quantities necessary for the calculations are given in terms of integrals of the partition functions. The particle methods are very flexible because they are mesh free. The basic idea in the FVPM is to incorporate elements of the classical FVM into a particle method. Therefore, concepts like the numerical flux functions and/or the treatment of the boundary

conditions are taken from the classical FVM. The solution procedure couples the flow solver with the structure and grid dynamics.

In the classical FVM the Euler equations written in an Arbitrary Lagrangian–Eulerian formulation are discretized using a Godunov type, cell–centered, finite volume method. The numerical fluxes are evaluated using Roe's flux–difference scheme. Solutions are advanced in time by a four–stage Runge-Kutta time–stepping scheme. At each time level, the displacements of the interior nodes are calculated with an iterative procedure, after imposing the displacements of the nodes located on the boundaries. Finally, the Geometric Conservation Law is used to compute the local cell volumes at the current time level.

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Thursday, 9:40–10:05

## Some estimates for the first eigenvalue of one Sturm–Liouville problem

Maria Telnova

**Abstract.** We consider the Sturm–Liouville problem

$$y'' - Q(x)y + \lambda y = 0, \quad x \in (0,1), \quad y(0) = y(1) = 0,$$

where $Q$ belongs to the set of real–valued locally integrable functions on $(0,1)$ with non–negative values such that the following integral condition holds:

$$\int_0^1 x^\alpha (1-x)^\beta Q^\gamma(x)\, dx = 1, \quad \alpha, \beta, \gamma \in \mathbb{R}, \quad \gamma \neq 0.$$

We estimate

$$m_{\alpha,\beta,\gamma} = \inf_{Q \in T_{\alpha,\beta,\gamma}} \lambda_1(Q), \quad M_{\alpha,\beta,\gamma} = \sup_{Q \in T_{\alpha,\beta,\gamma}} \lambda_1(Q).$$

Let $C_0^\infty(0,1)$ be the subset of functions in $C^\infty(0,1)$ that have supports compactly embedded in $(0,1)$. By $H_Q$ denote the closure of the set $C_0^\infty(0,1)$ in the norm

$$\|y\|_{H_Q}^2 = \int_0^1 \left(y'^2 + Qy^2\right)\,dx.$$

One can show that

$$\lambda_1(Q) = \inf_{y \in H_Q \setminus \{0\}} R[Q,y],$$

where

$$R[Q,y] = \frac{\int_0^1 \left(y'^2 + Q(x)y^2\right)\,dx}{\int_0^1 y^2\,dx}.$$

**Theorem.** *If $\gamma < 0$ or $0 < \gamma < 1$, then $M_{\alpha,\beta,\gamma} = \infty$. If $\gamma \geqslant 1$, then $M_{\alpha,\beta,\gamma} < \infty$. In particular, if $\gamma \geqslant 1$, $\alpha, \beta > \gamma$, then*

$$M_{\alpha,\beta,\gamma} = R\left[\frac{1}{y_1^2}, y_1\right], \quad y_1(x) = x^{\frac{\alpha}{2\gamma}}(1-x)^{\frac{\beta}{2\gamma}};$$

*if $\gamma > 1$, $\alpha, \beta \leqslant \gamma$ ($\alpha \leqslant \gamma < \beta \leqslant 2\gamma - 1$, $\beta \leqslant \gamma < \alpha \leqslant 2\gamma - 1$), then there are functions $Q_* \in T_{\alpha,\beta,\gamma}$ and $u \in H_{Q_*}$, such that*

$$M_{\alpha,\beta,\gamma} = R[Q_*, u] = \inf_{y \in H_{Q_*} \setminus \{0\}} \frac{\int_0^1 y'^2\,dx + \left(\int_0^1 |y|^{\frac{2\gamma}{\gamma-1}} \cdot x^{\frac{\alpha}{1-\gamma}}(1-x)^{\frac{\beta}{1-\gamma}}\,dx\right)^{\frac{\gamma-1}{\gamma}}}{\int_0^1 y^2\,dx};$$

*if $\gamma \geqslant 1$, $\alpha > 2\gamma - 1$, $\beta \leqslant \gamma$, then*

$$M_{\alpha,\beta,\gamma} \leqslant R\left[\frac{1}{y_2^2}, y_2\right], \quad y_2(x) = x^{\frac{\alpha}{2\gamma}}(1-x)^{\frac{1+\varepsilon}{2}};$$

*if $\gamma = 1 \geqslant \alpha \geqslant 0 > \beta$ ($\gamma = 1 \geqslant \beta \geqslant 0 > \alpha$), then $M_{\alpha,\beta,\gamma} \leqslant 2\pi^2$; if $\gamma = 1 \geqslant \alpha$, $\beta \geqslant 0$, then $M_{\alpha,\beta,\gamma} \leqslant 3\pi^2$; if $\gamma = 1$, $\alpha, \beta < 0$, then $M_{\alpha,\beta,\gamma} \leqslant \frac{5}{4}\pi^2$.*

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Tuesday, 9:40–10:05

## Variants of optimality criteria method for optimal design problems

Marko Vrdoljak

**Abstract.** We consider multiple state optimal design problems, aiming to find the best arrangement of two given isotropic materials, such that the obtained body has some optimal properties regarding $m$ different regimes. Speaking in terms of thermal conductivity, the cost function is an integral functional taking into account temperatures $u_1, \ldots, u_m$ (state functions) of the body, obtained for different right hand sides (given source terms) $f_1, \ldots, f_m$.

Using the homogenisation method as the relaxation tool, the standard variational techniques lead to necessary conditions of optimality. These conditions give the basis for the optimality criteria method, a commonly used numerical (iterative) method for optimal design problems. We study two variants of this method. The explicit calculation of the design update is possible for both of them, making the implementation much simpler and similar to the case of one state equation.

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Friday, 12:20–12:45

## The solving of finite-dimensional fixed point problem with adaptive algorithms based on Mann–Ishikawa iterations

Irina F. Yumanova

**Abstract.** One way to improve the convergence and extend the scope of applicability of linearly convergent iterative processes is to build processes of the Fejer type which are called Mann-Ishikawa iterations. Suppose that the problem of a fixed point is defined by the equality

$$\mathbf{x} = \mathbf{\Phi}(\mathbf{x}).$$

Here $\mathbf{\Phi} : \mathbb{R}^n \to \mathbb{R}^n$ is a given continuous nonlinear mapping. These iterations are based on the formula

$$\begin{aligned} \tilde{\mathbf{x}}^{(k+1)} &= \frac{\lambda_i^{(k)}}{1+\lambda_i^{(k)}}\tilde{\mathbf{x}}^{(k)} + \frac{1}{1+\lambda_i^{(k)}}\mathbf{x}^{(k+1)}, \\ \mathbf{x}^{(k+1)} &= \mathbf{\Phi}\left(\tilde{\mathbf{x}}^{(k)}\right) \end{aligned} \qquad k = 0, 1, \ldots, \quad i = 1, \ldots, n.$$

A careful study of the one-dimensional case shows that, to ensure fast convergence of the method, when fixing the parameter $\lambda^{(k)}$ at each iterative step, it is important to consider the nature of convergence/divergence of simple iterations delivering the next intermediate value $\mathbf{x}^{(k+1)}$. In an $n$-dimensional case, the behavior of the starting and intermediate points during each approximation can be similarly taken into account for each coordinate separately.

We use two options for calculating values $\lambda_i^{(k)}$. One way is the approach based on the Aitken $\Delta^2$ process. Another way is the approach based on the Wegstein method. We use the componentwise approach that shift tracking of approximations in each coordinate more subtle.

Testing the proposed algorithms was conducted on a large number of nonlinear systems. The transformation of systems in the form $\mathbf{F}(\mathbf{x}) = \mathbf{0}$ to a fixed point problem was performed using the formula

$$\mathbf{x} = \mathbf{x} - \left[\mathbf{F}'\left(\mathbf{x}^{(0)}\right)\right]^{-1}\mathbf{F}(\mathbf{x}),$$

which is the basis for the modified (simplified) Newton's method, the latter being a special case of simple iterations. In addition to the proposed algorithms, the systems were solved using the basic and modified Newton's methods. The results of testing of new methods characterize them as quite successful.

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Monday, 16:20–16:45

# Asymptotic equivalence of solutions to a generalization of the Lane–Emden equation

Sergey Zabolotskiy

**Abstract.** Consider two equations

$$y'' + \frac{n-1}{r}y' - |y|^k \operatorname{sign} y = 0, \tag{1}$$

$$y'' + \frac{n-1}{r}y' - |y|^k \operatorname{sign} y = f(r), \quad k,\ n \in \mathbb{R}, \quad r \geq 0. \tag{2}$$

**Theorem 1.** *Put $\beta = \frac{2}{k-1}$. Suppose $y(r)$ is a proper solution to equation (1) and $k > 1$ then the following statements are true:*

*a) if $n > \frac{2k}{k-1}$ then*

$$y(r) \sim Cr^{-n+2}$$

*as $r \to +\infty$ with some non-zero constant $C$. Such a solution exists for any $C \neq 0$;*

*b) if $n = \frac{2k}{k-1}$ then*

$$y \sim \pm \left(\frac{2}{(k-1)^2 r^2 \ln r}\right)^{\frac{\beta}{2}}$$

*as $r \to +\infty$. A solution with such asymptotic behaviour exists.*

*c) if $n < \frac{2k}{k-1}$ then*

$$y(r) = \pm \left(\beta(\beta+1) - \beta(n-1)\right)^{\frac{\beta}{2}} r^{-\beta} + o(r^{-\beta})$$

*as $r \to +\infty$. A solution with such asymptotic behaviour exists.*

**Theorem 2.** *Let $k > 1$ and $f(r)$ be a function such that*

$$\int_0^{+\infty} |f(r)|^2 e^{2br}\, dr < \infty$$

*for some $b > 0$. Then for any proper solution to equation (2) with $k > 1$ such that this solution and its derivative tend to zero as $r \to +\infty$ there exists the unique proper solution $\tilde{y}(r)$ to equation (1) with $k > 1$ satisfying the following conditions:*

$$|y(r) - \tilde{y}(r)| = o(e^{-br}), \qquad \int_0^{+\infty} |y(r) - \tilde{y}(r)|^2 e^{2br} dr < \infty.$$

## References

[1] I. V. Astashova, *On asymptotic behavior of solutions to a quasi–linear second order differential equation*, Functional differential equations **16** (2009), no. 1, 93–115.

[2] Yu. V. Egorov, V. A. Kondratiev, O. A. Oleinik, *Asymptotic behavior of the solutions to nonlinear elliptic and parabolic systems in tube domains*, Mat. Sb. **198** (1998), no. 3, 45–68.



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Tuesday, 11:20–11:45

## Homogenization in periodic porous domains


Rachad Zaki



**Abstract.** The periodic unfolding method is a recent technique for homogenization. It was first introduced for fixed domains in order to treat classical periodic homogenization problems [1], and was later extended to periodically perforated domains [2]. The method allows to overcome several obstacles one would face with homogenization like regularity hypotheses on the boundary of the domain that are needed for defining extension operators, and it also allows to treat the case when the unit hole is not a compact subset of the open unit cell, with some additional conditions. The periodic unfolding method is based on two main ingredients: one or more unfolding operators, depending on the problem, and a macro-micro decomposition of functions using two special operators. We present the periodic unfolding method in periodic porous media and show several applications using different types of boundary conditions.



## References

[1] D. Cioranescu, A. Damlamian, and G. Griso, *The periodic unfolding method in homogenization*, SIAM J. Math. Anal. **40** (2008), 1585–1620.

[2] D. Cioranescu, A. Damlamian, P. Donato, G. Griso, and R. Zaki, *The periodic unfolding method in domains with holes*, SIAM J. Math. Anal. **44** (2012), 718–760.

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